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FOREIGN TECHNOLOGY DIVISION

OPTIMIZATION OF SPACE TRAJECTORIES

By

S. N. Kirpichnikov

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OPTIMIZATION OF SPACE TRAJECTORIES

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Block	Italic	Transliteration	Block	Italic	Transliteration
А а	*А а*	A, a	Р р	*Р р*	R, r
Б б	*Б б*	B, b	С с	*С с*	S, s
В в	*В в*	V, v	Т т	*Т т*	T, t
Г г	*Г г*	G, g	У у	*У у*	U, u
Д д	*Д д*	D, d	Ф ф	*Ф ф*	F, f
Е е	*Е е*	Ye, ye; E, e*	Х х	*Х х*	Kh, kh
Ж ж	*Ж ж*	Zh, zh	Ц ц	*Ц ц*	Ts, ts
З з	*З з*	Z, z	Ч ч	*Ч ч*	Ch, ch
И и	*И и*	I, i	Ш ш	*Ш ш*	Sh, sh
Й й	*Й й*	Y, y	Щ щ	*Щ щ*	Shch, shch
К к	*К к*	K, k	Ъ ъ	*Ъ ъ*	"
Л л	*Л л*	L, l	Ы ы	*Ы ы*	Y, y
М м	*М м*	M, m	Ь ь	*Ь ь*	'
Н н	*Н н*	N, n	Э э	*Э э*	E, e
О о	*О о*	O, o	Ю ю	*Ю ю*	Yu, yu
П п	*П п*	P, p	Я я	*Я я*	Ya, ya

*ye initially, after vowels, and after ъ, ь; e elsewhere.
When written as ё in Russian, transliterate as yё or ё.

RUSSIAN AND ENGLISH TRIGONOMETRIC FUNCTIONS

Russian	English	Russian	English	Russian	English
sin	sin	sh	sinh	arc sh	$\sinh^{-1}$
cos	cos	ch	cosh	arc ch	$\cosh^{-1}$
tg	tan	th	tanh	arc th	$\tanh^{-1}$
ctg	cot	cth	coth	arc cth	$\coth^{-1}$
sec	sec	sch	sech	arc sch	sech^{-1}
cosec	csc	csch	csch	arc csch	csch^{-1}

Russian	English
rot	curl
lg	log

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Section 1

OPTIMIZATION OF SPACE TRAJECTORIES

S. N. Kirpichnikov.

The works of this section are dedicated to the development of methods of the mechanics of optimal controllable systems as applied to problems of space dynamics. The primary number of works is connected with the development of analytical methods of the optimum selection of transfer orbits in a central field and with the approximate solution of separate model problems.

The work of V. A. Antonov and A. S. Shmyrov, placed in the section of brief scientific reports, is directly adjacent to the articles of this section.

OPTIMIZATION OF DESCENT MANEUVERS OF SATELLITE MODULE FROM NEAR-PLANET ORBIT

Let us examine a space vehicle, moving along an elliptical orbit in the gravitational field of a spherically symmetric planet. The space vehicle consists of two parts: module descendable to the planet (module I) and orbital module (module II). It is required to construct an optimum, with respect to fuel consumption, descent maneuver of module I to the planet. The maneuver is executed with the aid of a single initial pulse, applied to module I so that the orbital module continues to move along the initial orbit, and the descent module transfers to the approach trajectory to the planet. The landing maneuver of module I is terminated after entry into the dense layers of the atmosphere and their passage. The planet may not have an atmosphere, then the maneuver is terminated by the hard contact of module 1 with its surface. In the latter case it is necessary everywhere below to formally equate the altitude of the atmosphere to zero.

Let us agree by start to mean the moment of the initial pulse. By finish for determinacy we will mean the landing of module I on the surface of the planet. However, all the discussed stops being valid

if as the moment of finish we select some other fixed moment between entry of module I into dense layers of the atmosphere and its landing.

The initial orbit of the space vehicle and the trajectory of the descent module right up to its entry into the dense layers of the atmosphere are considered Keplerian, where there are considered only elliptical orbits of descent with straight motion with respect to the initial orbit. It is assumed that the angular range, change of altitude and the time of motion on the segment of passage of dense layers of the atmosphere to the finish point are known beforehand, and that this segment is located in planetocentric plane, passing through the velocity vector of module I during its entry into the dense layers of the atmosphere.

Taking into account the Tsiolkovskiy formula, we will minimize the characteristic velocity of the initial pulse, i.e., the modulus of pulse change of velocity of module I.

In this work the formulated problem is investigated with complicated boundary conditions. Namely limitations are introduced on the angle of entry of the descent module and on the distance between the modules at the moment of finish. Furthermore, there is considered the condition of direct visibility between the modules and more

general limitation on the co-altitude of module II at the point of finish, and there can be taken into account the limitation on the velocity of module I at the moment of entry.

Following the method proposed by Ting Lu [1] it is easy to prove that the optimum descent orbits of the maneuvers examined here in all cases, being of practical interest, should lie in the plane of the initial orbit; therefore this investigation is limited to coplaner formulation of the problem.

§1. Mathematical Formulation of the Problem and General Conclusions.

In the plane of motion let us introduce polar coordinates r, ϑ with the origin at the center of the planet so that the direction of positive reading of angle ϑ coincides with the direction of motion along the initial orbit.

Along with Keplerian elements of orbits: large semiaxis a, eccentricity e and angular distance ω of pericenter from the polar axis we will consider elements p, q, introduced by formulas

$$p=\frac{1}{\sqrt{a(1-e^2)}},\quad q=\frac{e}{\sqrt{a(1-e^2)}}. \qquad (1)$$

Let us assume p_1, q_1, a_1, e_1, ω_1 - parameters of initial orbit, and p, q, a, e, ω - parameters of intermediate orbit of descent module.

Let us designate moments of time and polar coordinates of the points of start and entry of module I through t_1, r_1, ϑ_1 and t_2, r_2, ϑ_2 respectively. Obviously

$$r_2 = r_{пл} + h_{атм}, \tag{2}$$

where $r_{пл}$ - radius of planet, $h_{атм}$ - height of dense layers of the atmosphere. Let us introduce parameter p_2 by formula

$$p_2 = \frac{1}{\sqrt{r_2}}. \tag{3}$$

It is assumed that change of Δr polar radius, angular range $\Delta\vartheta$ and duration Δt of the segment of flight of module I in dense layers of the atmosphere to the point of finish are known beforehand. Therefore time $\tilde{t}_2$ of finish and polar coordinates $\tilde{r}_2$, $\tilde{\vartheta}_2$ of the point of finish will be

$$\left.\begin{aligned} \tilde{r}_2 &= r_2 - \Delta r, \\ \tilde{\vartheta}_2 &= \vartheta_2 + \Delta\vartheta, \\ \tilde{t}_2 &= t_2 + \Delta t. \end{aligned}\right\} \tag{4}$$

If by finish we mean the landing of module I on the surface of the planet, then

$$\Delta r = h_{атм}, \quad \tilde{r}_2 = r_{пл}, \tag{5}$$

if the entry of this module, then

$$\Delta t = \Delta r = \Delta\vartheta = 0, \quad \tilde{r}_2 = r_2, \quad \tilde{\vartheta}_2 = \vartheta_2, \quad \tilde{t}_2 = t_2. \tag{6}$$

For polar radii r_1 and r_2 we have

$$r_1=[p_1^2+p_1q_1\cos(\vartheta_1-\omega_1)]^{-1}=[p^2+pq\cos(\vartheta_1-\omega)]^{-1}, \qquad (7)$$

$$r_2=p_2^{-2}=[p^2+pq\cos(\vartheta_2-\omega)]^{-1}. \qquad (8)$$

Characteristic velocity ΔU of initial pulse can be reduced to the form [2]

$$\Delta U=K\Delta V, \qquad (9)$$

$$\Delta V=\left\{q_1^2+3p_1^2+q^2-p^2-\frac{2p_1^3}{p}-2qq_1\cos(\omega_1-\omega)-\right.$$
$$\left.-\frac{2(p-p_1)^2q_1}{p}\cos(\vartheta_1-\omega_1)\right\}^{\frac{1}{2}}, \qquad (10)$$

where K - Gaussian constant, multiplied by the square root of the planet mass. For value ΔV, distinguished only by constant multiplier from ΔU, let us retain the name characteristic velocity.

Thrust angle Φ_T during the initial thrust we will read in reverse motion of the direction from positive transversal to the direction of pulse, then we will have

$$\operatorname{tg}\Phi_T=\frac{p\,[q\sin(\vartheta_1-\omega)-q_1\sin(\vartheta_1-\omega_1)]}{(p_1-p)\,[p_1+q_1\cos(\vartheta_1-\omega_1)]}. \qquad (11)$$

The signs of the numerator and denominator in the right side of (11) coincide respectively with signs $\sin\Phi_T$ and $\cos\Phi_T$.

Let us determine angle Φ of entry as the angle between the velocity vector of the descent module and the plane of local horizon at moment of time t_2:

$$\operatorname{tg}\Phi=-\frac{U_r}{U_\vartheta}=-\frac{pq\sin(\vartheta_2-\omega)}{p_2^2}, \quad \Phi\in\left[0,\frac{\pi}{2}\right), \qquad (12)$$

where U_r and U_ϑ - the radial and transversal components of velocity

$\bar{U}_{вх}$ of module I respectively at the moment of its entry. Values $U_r, U_\vartheta, U_{вх}$ are determined by formulas

$$U_r = Kq\sin(\vartheta_2-\omega),\quad U_\vartheta = \frac{Kp_2^2}{p}, \tag{13}$$

$$U_{вх} = K\sqrt{2p_2^2+q^2-p^2}. \tag{14}$$

Let us assume at moment of time $\tilde{t}_2$ the orbital module has polar radius r_3 and polar angle ϑ_3. For the last values we find

$$\int\limits_{\vartheta_1-\omega}^{\vartheta_2-\omega} \frac{dv}{p(p+q\cos v)^2} + K\Delta t = \int\limits_{\vartheta_1-\omega_1}^{\vartheta_3-\omega_1} \frac{dv}{p_1(p_1+q_1\cos v)^2}. \tag{15}$$

$$r_3 = [p_1^2+p_1q_1\cos(\vartheta_3-\omega_1)]^{-1}. \tag{16}$$

Distance l between modules at the moment of finish is equal to

$$l = [r_3^2+\tilde{r}_2^2-2\tilde{r}_2r_3\cos(\vartheta_3-\tilde{\vartheta}_2)]^{\frac{1}{2}}, \tag{17}$$

and co-altitude z of module II at the point of finish will be

$$z = \arccos\frac{r_3^2-l^2-\tilde{r}_2^2}{2\tilde{r}_2 l}. \tag{18}$$

The relationships obtained above show that as the basis parameters, determining the descent maneuver, there can be obtained

$$p,\ q,\ \vartheta_1,\ \vartheta_2,\ \vartheta_3,\ \omega, \tag{19}$$

which are dependent and satisfy the following connections:

$$\varphi_1 = p^2+pq\cos(\vartheta_1-\omega)-p_1^2-p_1q_1\cos(\vartheta_1-\omega_1)=0, \tag{20}$$

$$\varphi_2 = p^2+pq\cos(\vartheta_2-\omega)-p_2^2=0, \tag{21}$$

$$\varphi_3 = \int\limits_{\vartheta_1-\omega}^{\vartheta_2-\omega} \frac{dv}{p(p+q\cos v)^2} + K\Delta t - \int\limits_{\vartheta_1-\omega_1}^{\vartheta_3-\omega_1} \frac{dv}{p_1(p_1+q_1\cos v)^2} = 0. \tag{22}$$

By the sought optimum descent maneuver we will mean the maneuver, to which corresponds the least value of characteristic velocity ΔV.

Let us turn to the calculation of additional limitations. Let us

examine first only the limitations on variables l and Φ. Angle Φ of entry should lie in some preset interval:

$$\Phi_{min} < \Phi < \Phi_{max}. \tag{23}$$

Distance l between modules at the moment of finish should not exceed the fixed maximum value of L:

$$l \leqslant L. \tag{24}$$

By introducing additional real variables α, β, conditions (23), (24) let us rewrite so:

$$\varphi_4 = (\Phi_{min} - \Phi)(\Phi - \Phi_{max}) - \alpha^2 = 0, \tag{25}$$

$$\varphi_5 = l - L + \beta^2 = 0. \tag{26}$$

Thus, with conditions (23), (24) the problem is mathematically reduced to the minimization of function (10) in the total of real variables

$$p,\ q,\ \vartheta_1,\ \vartheta_2,\ \vartheta_3,\ \omega,\ \alpha,\ \beta, \tag{27}$$

which satisfy conditions (20)-(22), (25), (26). As is known, the derivatives with respect to all variables (27) from the Lagrange function should be equal to zero

$$\Delta V + \sum_{i=1}^{5} \lambda_i \varphi_i, \tag{28}$$

where λ_1, λ_2, ..., λ_5 - unknown constant multipliers.

Analysis of the equation of extremum, cooresponding to variable β, shows that either this variable is equal to zero and condition $l=L$ is fulfilled, or the sought solution corresponds to the relative minimum of function ΔV in the auxiliary problem, in which conditions (24), (26) are dropped. Therefore it is possible to recommend such a

sequence off investigation.

1. The problem is examined without taking into account limitations (23)-(26). Using the results of [2] and the method, similar to that developed in article [3], it is easy to show that single steady-state solutions in this case are the solutions obtained in [2] during the investigation of energetically optimum single-pulse flight between elliptical initial and circular final orbits. The indicated solutions are characterized by apsis tangential connections of intermediate trajectories with initial orbit and circle of radius r_2. The trajectory, exiting the apocenter of the initial orbit, always requires less fuel consumption in comparison with the trajectory exiting the pericenter. For both steady-state solutions there is fulfilled equality

$$\Phi=0. \tag{29}$$

In general case point (29) lies outside the interval (23), and it is necessary to change to the search for all relative minimums of function ΔV, having kept only conditions (20)-(22) and having fixed angle Φ:

$$\Phi=\Phi_{min}. \tag{30}$$

The analysis and solution of this problem with any fixed angle Φ are contained in [3].

If among all the solutions obtained at this stage there are

those, for which inequality (24) is satisfied, then by comparison of the characteristic velocities corresponding to them the sought solution is found. In the opposite case, when condition (24) is disturbed for all solutions, the sought optimum maneuver is characterized by equality

$$l=L, \tag{31}$$

and it is necessary to change to point 2.

2. A problem is examined in which conditions (20)-(22), (23) are kept, and inequality (24) is replaced by equality (31). For the numerical investigation there can be used the method developed in the following paragraph for the determination of the optimum descent maneuver at prescribed values of l and Φ. Interval $[\Phi_{min}, \Phi_{max}]$ is broken down with the sequence of points into a row of equal parts and for each point there is calculated the optimum maneuver. Then by comparison of functions ΔV there are found the approximate values of angle Φ and the parameters of the sought optimum maneuver. If the obtained accuracy is insufficient, there can be conducted local refinement of these approximate values by one of the methods of successive approximations.

Let us assume now, besides conditions (20)-(24), limitations are introduced on variables z and $U_{вх}$. The problem of optimization of function ΔV, just as in point 2, does not lend itself to analytical

investigation. For its numerical solution there can be used the method developed in the following paragraph for the construction of optimum descent maneuver of module I with fixed parameters l and Φ, which makes it possible to consider the indicated limitations on variables z and U_m. Determination of the sought solution is performed similar to that discussed above in point 2. The distinction is only that here into a row of parts are broken down two intervals $[\Phi_{min}, \Phi_{max}]$, $[a_1(1-e_1)-\tilde{r}_2, L]$, and the calculation is performed for all pairs of points, where one point is taken from the interval of change of angle Φ, and the other - from the interval of allowable values of distance l. The indicated approach is especially suitable when high accuracy is not required, and it is important to obtain a picture of the change of parameters of optimum maneuvers depending on the variations of values l and Φ.

§2. Mathematical Algorithm of Construction of Optimum Descent Maneuver With Fixed Values of l and Φ

Below is presented a description of the method of solution of the examined problem in the assumption that there are assigned angle Φ of entry of descent module and distance l between the modules at the moment of finish. Mathematically it is required to find the

smallest value of function (10) in the total of variables (19), which satisfy conditions (12), (17), (20)-(22), where parameters l and Φ are considered known. The given method makes it possible to consider additionally the limitations on variables z and U_{BX}.

We will consider that the initial orbit is not circular, where there are fulfilled inequalities

$$0<q_1<p_1, \qquad (32)$$
$$r_{1\pi}>r_2, \qquad (33)$$
$$r_{1\pi}<l+\tilde{r}_2, \qquad (34)$$
$$r_{1a}>l-\tilde{r}_2, \qquad (35)$$

where $r_{1\pi}=a_1(1-e_1)$ and $r_{1a}=a_1(1+e_1)$ - the distances of the pericenter and apocenter respectively of the initial orbit. Condition (33) signifies that the initial orbit is located entirely outside the dense layers of the atmosphere.

If $r_{1a}<l-\tilde{r}_2$ or $r_{1\pi}>l+\tilde{r}_2$, then obviously the descent maneuvers with given distance l are generally impossible. Cases

$$\left.\begin{array}{l} q_1=0, \\ r_{1a}=l-\tilde{r}_2, \\ r_{1\pi}=l+\tilde{r}_2 \end{array}\right\} \qquad (36)$$

will not be examined here, since for each of them the problem in question loses its extremal character and is transformed into determinate.

Bet us change to discussion of the method. From equalities (12),

(21) we obtain

$$q=\left[\frac{p_2^4\sec^2\Phi}{p^2}-2p_2^2+p^2\right]^{1/2}, \tag{37}$$

$$\operatorname{tg}(\vartheta_2-\omega)=\frac{-p_2^2\operatorname{tg}\Phi}{p_2^2-p^2}, \tag{38}$$

where the signs of the numerator and denominator in the last formula coincide with the signs of $\sin(\vartheta_2-\omega)$ and $\cos(\vartheta_2-\omega)$ respectively.

Relationship (13) makes it possible to find

$$\vartheta_2=\vartheta_3-\Delta\vartheta-\gamma_1\arccos\frac{r_3^2+\tilde{r}_2^2-l^2}{2r_3\tilde{r}_2},\quad \gamma_1=\pm 1, \tag{39}$$

and from equality (20) we have

$$\left.\begin{aligned}\sin(\vartheta_1-\omega_1)&=\frac{-b_2b_3+\gamma_2b_1\sqrt{b_1^2+b_2^2-b_3^2}}{b_1^2+b_2^2},\\ \cos(\vartheta_1-\omega_1)&=\frac{-b_1b_3-\gamma_2b_2\sqrt{b_1^2+b_2^2-b_3^2}}{b_1^2+b_2^2},\\ \gamma_2&=\pm 1,\end{aligned}\right\} \tag{40}$$

where

$$\left.\begin{aligned}b_1=pq\cos(\omega-\omega_1)-p_1q_1,\quad b_2=pq\sin(\omega-\omega_1),\\ b_3=p^2-p_1^2.\end{aligned}\right\} \tag{41}$$

Subsequently by symbols q, ϑ_1, ϑ_2, ω we will mean the functions of parameters p, ϑ_3, determined by relationships (37)-(41). With such elimination of variables the remaining unknowns p, ϑ_3 should satisfy the following inequalities:

$$0<q<p, \tag{42}$$

$$|\tilde{r}_2^2+r_3^2-l^2|\leqslant 2r_3\tilde{r}_2, \tag{43}$$

$$b_1^2+b_2^2-b_3^2>0. \tag{44}$$

Due to dependence (42), i.e., the assumption about ellipticity of transfer orbit, we reduce condition (22) to the form

$$\Delta=0, \tag{45}$$

where

$$\begin{aligned}\Delta=K\Delta t+a^{\frac{3}{2}}[E_2-E_1-e(\sin E_2-\sin E_1)]-\\ -a_1^{\frac{3}{2}}[E_2^{(1)}-E_1^{(1)}-e_1(\sin E_2^{(1)}-\sin E_1^{(1)})].\end{aligned} \tag{46}$$

where E_1, E_2 and $E_1^{(1)}, E_2^{(1)}$ - eccentric anomalies of modules I and II at moments t_1, t_2 respectively. The eccentric anomalies are easily computed by known formulas of elliptic motion. Time t_2-t_1 of motion before entry is computed so:

$$t_2-t_1=\frac{a^{\frac{3}{2}}}{K}[E_2-E_1-e(\sin E_2-\sin E_1)]. \quad (47)$$

Let us derive now some inequalities and estimations, which are used during numerical solution. First of all from condition (42) and dependence

$$r_a > r_{1n}, \quad (48)$$

where r_a - planetocentric distance of apocenter of transfer orbit, we find

$$p_* < p < p_{**}, \quad (49)$$

where

$$p_* = \frac{p_2 \sec\Phi}{\sqrt{2}}, \quad (50)$$

$$p_{**} = \max\left\{\sqrt{\frac{p_2^2\sec^2\Phi-(p_1^2+p_1q_1)^2}{2[p_2^2-p_1^2-p_1q_1]}},\ \sqrt{p_1^2+p_1q_1}\right\}. \quad (51)$$

Let us note that here the necessary and sufficient condition (44) of intersection of intermediate and initial orbits is replaced by necessary condition (48), therefore subsequently during the solution it is necessary to consider condition (48).

Dependence (43) and obvious inequality

$$r_{1н} \leqslant r_3 \leqslant r_{1в} \quad (52)$$

will give

$$r_* \leqslant r_3 \leqslant r_{**}. \quad (53)$$

where

$$r_* = \max\{r_{1н}, l - \tilde{r}_2\}, \quad r_{**} = \min\{r_{1в}, l + \tilde{r}_2\}, \quad (54)$$
$$r_{**} > r_*. \quad (55)$$

The last inequality takes place due to relationships (34), (35).

Condition (53) determines two intervals, in which lie the sought values of angle ϑ_3:

$$\vartheta_3^{(1)} \leqslant \vartheta_3 \leqslant \vartheta_3^{(2)}, \quad (56)$$
$$2\pi - \vartheta_3^{(2)} \leqslant \vartheta_3 \leqslant 2\pi - \vartheta_3^{(1)}, \quad (57)$$

where

$$\left.\begin{aligned} \vartheta_3^{(1)} &= \omega_1 + \arccos \mu_1, \\ \vartheta_3^{(2)} &= \omega_1 + \arccos \mu_2, \end{aligned}\right\} \quad (58)$$

$$\mu_1 = \frac{\frac{1}{r_*} - p_1^2}{p_1 q_1}, \qquad \mu_2 = \frac{\frac{1}{r_{**}} - p_1^2}{p_1 q_1}, \quad (59)$$

where obviously

$$\mu_1 > \mu_2, \quad \mu_1 < 1, \quad \mu_2 > -1. \quad (60)$$

The developed method permits taking into calculation the limitations on variables z and $U_{вх}$. Let us assume there is introduced requirement

$$z \leqslant z_{max}, \quad (61)$$

where z_{max} - assigned maximum value of co-altitude of module II at the point of finish at moment $\tilde{t}_2$. Then from formulas (18), (61) we obtain

$$r_3 > r_{3*}, \quad (62)$$
$$r_{3*} = \sqrt{l^2 + \tilde{r}_2^2 + 2\tilde{r}_2 l \cos z_{max}}. \quad (63)$$

The calculation of condition (62) is reduced now to computation of r_* in dependences (53)-(59) by formula

$$r_* = \max\{r_{1*},\ l-\tilde{r}_2,\ r_{3*}\}. \qquad (64)$$

In a particular case, when limitation (61) is a condition of direct visibility between modules at the moment of finish, distance r_{3*} becomes equal to

$$r_{3*} = \sqrt{l^2+\tilde{r}_2^2}. \qquad (65)$$

Finally, limitation on velocity $U_{вх}$ we take in the form

$$U_{вх} \leqslant U, \qquad (66)$$

where constant U is assigned. If it is possible to disregard the speed of rotation of the atmosphere in comparison with value $U_{вх}$, then condition (66) is equivalent to the requirement that the entry velocity of module I not exceed some fixed value of U.

From relationships (14), (37), (66) we find

$$p > \tilde{p},\quad \tilde{p} = \frac{K p_2^2 \sec\Phi}{U},\quad \tilde{p} > p_*, \qquad (67)$$

and, consequently, condition (66) will be considered if inequality (49) is replaced by

$$\tilde{p} < p < p_{**}. \qquad (68)$$

Subsequently, if we introduce one or both limitations (62), (66), we will consider that the corresponding changes in inequalities (49), (53)-(59) are produced.

Let us formulate the obtained results. The problem of optimization was reduced to the search for parameter p from interval (49) and angle ϑ_3 in one of regions (56), (57) so that conditions (44), (45) would be fulfilled, and function ΔV would have the smallest value. It is convenient as the variable during optimization to select parameter p, and to consider corresponding angle ϑ_3 as the radical of equation (45) lying in region (56), (57), for which condition (44) is fulfilled.

Depending on the concrete selection of parameters γ_1, γ_2 four types of maneuvers should be investigated. Let us introduce value

$$j=\frac{5-2\gamma_1-\gamma_2}{2},\quad \gamma_1=\pm 1,\quad \gamma_2=\pm 1, \qquad (69)$$

which takes values j = 1, 2, 3, 4 for these types. The descent trajectories for j = 1, 3 (γ_2=1) differ from the trajectories for j = 2, 4 (ϑ_2= -1) by concrete selection of the point of start in one of the two points of intersection of initial and intermediate orbits. Therefore in one case the flight path of module I will emerge on the initial segment from the region limited by the initial orbit, and in the other case will entirely lie inside the indicated region. Further, maneuvers j = 1, 2 (ϑ_1=1) differ from maneuvers j = 3, 4 (ϑ_1= -1) by the fact that at the moment of finish in the first case the orbital module leads the descent, i.e., has large polar angle. In

the second case, conversely, the polar angle of the descent module is larger than the polar angle of the orbital. Let us note that with such comparison angles $\tilde{\vartheta}_2, \vartheta_3$ should be brought to interval $|\tilde{\vartheta}_2-\vartheta_3|\leqslant\pi$.

Let us present a brief description of a computer program, using the algorithm indicated above.

I. A search is conducted for the approximate desired value of parameter p by means of the total survey of interval (49), which is broken down into n equal parts. For each point of division there are found all the radicals of equation (45), which lie in regions (56), (57) and for which is fulfilled condition (44). During the computing of radicals there are taken rough constants of accuracy, which provides the quickness of operation of this program unit. From the multitude of all the sorted out values of parameter p and the radicals corresponding to them there is selected pair p, ϑ_3, which corresponds to the smallest value of function ΔV. These quantities are taken as approximate optimum values.

II. There is found the exact optimum value of variable p and other parameters of the sought maneuver. Let us note that quantity ΔV as a function of parameter p with optimum selection of radical ϑ_3 can have discontinuities of the first type and regions of unimportance,

where equation (45) does not have radicals of the required type at all. Due to the indicated special character of the problem it is expedient to refine the optimum values of parameters until the obtaining of certain accuracy by the method of successive approximations. In this case in each approximation the interval between two values of parameter p, adjacent to the optimum value of the preceding approximation, is broken down into some preset number of parts and calculation is performed for all the points of division. By comparison of the corresponding characteristic velocities the optimum value of parameters of the given approximation is determined. During computation of the radicals of function Δ there are taken constants, providing the assigned accuracy of computations.

In conclusion let us elaborate on the procedure of computation of radicals of function (46) for any fixed value of parameter p. For determinacy we are limited by descent maneuvers, for which the overall time $\tilde{t}_2-t_1$ of the maneuver is exactly less than the period of motion along the initial orbit. In accordance with this we will compute the eccentric anomalies, entering the equalities (45)-(47), taking into account conditions

$$\left.\begin{aligned} E_1+2\pi>E_2>E_1, \\ E_1^{(1)}+2\pi>E_1^{(1)}+2\pi-\delta>E_2^{(1)}>E_1^{(1)}, \end{aligned}\right\} \quad (70)$$

where δ - any assigned small quantity.

Regions (56), (57) are divided into N equal parts and function Δ

is computed successively for all the points of division.

Let us indicate one special feature of the given method. For any value of angle ϑ_3 first there is determined the geometric picture of the maneuver, i.e., mutual location of orbits, and then there are selected the specific legs of flight of the modules along orbits from the condition of fulfillment of inequalities (70). Therefore critical values of angle ϑ_3 can exist, which correspond to discontinuity of function E_2-E_1, if switching of the leg of flight occurred on the descent orbit, and functions $E_2^{(1)}-E_1^{(1)}$, if switching occurred on the trajectory of motion of module II. At the points of intersection one of the limiting values of the corresponding functions is equal to zero, and the amount of discontinuity is equal to 2π. It is easy to see that the radicals of function Δ can lie only at finite distance from the critical points. Let us select number N so that the variation of functions E_2-E_1, $E_2^{(1)}-E_1^{(1)}$ on each segment of subdivision of intervals (56), (57) does not exceed some constant A. The variation of function on the segments containing critical points will then be not less than $2\pi-A$. By increasing N and selecting constant A, it is always possible to achieve fulfillment of condition $A<2\pi-A$ and thereby obtain criterion, which makes it possible by the magnitude of fluctuation of function Δ to judge whether switching was inside the considered segment or not.

Let us return to the description of the procedure of computation of radicals. If on the boundaries of some segment of division of the intervals (56), (57) function Δ has different signs, and its fluctuation is less than constant A, then within this segment is found the radical of function Δ, which is computed with the aid of a series of interpolations.

Let us elaborate more on one specific feature of the algorithm. If for some value of ϑ_3 condition (44) is disturbed, then we will say that this value is found in the region of nonexistence of function Δ. When during sorting out of points from intervals (56), (57) we fall in the region of nonexistance of function Δ, then by successive division of the given segment in half we find the boundary of the region of nonexistence. Then we investigate the region, one of the boundaries of which is the last point during sorting out of points of subdivision of intervals (56), (57), and the other - the boundary of the region of nonexistence. With the fulfillment of conditions of the presence of radical in the indicated region the radical is computed by a series of interpolations. An analogous procedure is performed when from the region of nonexistence functions Δ transfer to the values of angle ϑ_3, for which condition (44) is fulfilled.

After all the radicals of function Δ are found for the parameter p in question, from them is selected a radical, which the smallest

value of function ΔV corresponds to. With this the determination of the radicals of function Δ is finished.

§3. Numerical Example

As an example of the use of the algorithm developed in §2 let us examine the problem of sending from a space vehicle, moving along elliptical orbit of a Venus satellite, probe (module I) for investigation of the upper layers of the Venus atmosphere. We will minimize the fuel consumption.

Let us select the following elements of the initial orbit of the space vehicle:

$$a_1 = 10000 \text{ км}, \quad e_1 = 0.28, \quad \omega_1 = 0°. \tag{71}$$

The height of pericenter of the orbit above the surface of Venus will be 1000 km, and the height of apocenter - 6600 km.

For numerical values of constant K and r_2 let us take

$$K^2 = 3.2423 \cdot 10^5 \text{ км}^3/\text{см}^2, \quad r_2 = 6200 \text{ км}. \tag{72}$$

Let us assume further that the moment of finish coincides with the moment of entry, then

$$\Delta r_2 = \Delta t = \Delta \vartheta = 0. \tag{73}$$

Bet us request that at the moment of finish values l and Φ had assigned values and the condition of direct visibility between modules was fulfilled.

Calculations were conducted on an M-20 computer. Two values of distance were selected

$$l=1200\ \text{км}, \quad l=1600\ \text{км} \tag{74}$$

and the interval of change of angle Φ was examined with pulse duty factor 5° from zero to 30°. The results of calculations are presented in Tables 1 and 2.

Φ	j	a, км (a)	e	ϑ_1	ϑ_2	ϑ_3	ω	ΔU, км/сек (b)	Φ_T	t_2-t_1, сек (c)	z	U_{bx}, км/сек (b)
0°	3	9497,4	0,3472	184°,99	368°,61	2°,95	8°,61	0,348	126°,14	5305	36°,03	8,394
5	3	9170,7	0,3343	222,83	355,63	350,25	15,74	0,329	149,09	3259	34,32	8,321
10	3	8949,2	0,3488	228,52	339,73	336,40	19,59	0,403	156,54	2787	20,80	8,268
15	1	8394,2	0,3616	245,54	330,91	331,27	31,62	0,601	163,91	2055	2,21	8,122
20	2	7486,4	0,3782	281,72	331,32	333,69	56,05	1,080	195,98	1045	14,73	7,828
25	2	[illegible]	[illegible]	301,31	332,70	335,88	73,93	1,773	214,39	645	19,83	7,634
30	[illegible]	[illegible]	[illegible]	311,70	334,02	337,69	85,84	2,480	222,28	468	22,96	7,526

Table 1. Parameters of optimum maneuvers for l = 1200 km.

Key: (a) km. (b) km/s. (c) s.

Φ	j	(a) a, км	e	ϑ_1	ϑ_3	ϑ_2	ω	(b) ΔU, км/сек	Φ_T	t_2-t_1, сек	z	(c) $U_{кх}$, км/сек
0°	3	9298,9	0,3333	219°,64	372°,46	361°,74	12°,46	0,268	156°,68	3589	56°,84	8,350
5	3	9321,2	0,3448	214,01	350,65	340,66	10,29	0,261	167,25	3491	52,26	8,355
10	3	9172,4	0,3633	212,90	333,36	325,39	11,92	0,328	167,40	3276	40,44	8,321
15	3	8888,3	0,3903	216,58	319,84	315,04	16,38	0,453	166,52	2918	23,71	8,253
20	1	8459,2	0,4242	222,85	310,42	310,42	24,15	0,655	162,30	2553	1,02	8,140
25	2	7698,6	0,4580	249,41	309,20	312,42	41,54	1,003	180,46	1580	15,80	7,904
30	2	7026,9	0,5103	270,74	310,54	315,59	59,02	1,552	197,20	999	24,99	7,645

Table 2. Parameters of optimum maneuvers for l = 1600 km.

Key: same as Table 1.

Let us note that for both values of distance l the minimum of characteristic velocity as a function of angle Φ is reached for some values of angle $\Phi \in (0°,5)$, where this velocity rapidly rises for $\Phi > 10°$. It is interesting to note also that velocity $U_{кх}$ for angles Φ, not exceeding 10-15° for optimum maneuvers, barely depends on angle Φ and is equal to approximately 8.3 km/s.

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